Approximate Probabilistic Power Flow

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Abstract. Power flow analysis is a necessary tool for operating and planning Power systems. This tool uses a deterministic approach for obtaining the steady state of the system for a specified set of power generation, loads, and network conditions. However this deterministic methodology does not take into account the uncertainty in the power systems, for example variability the in power generation, variation in the demand, changes in network configuration. The probabilistic power flow (PPF) study has been used as an useful tool to consider the system uncertainties in power systems. In this paper, we propose another alternative for solving the PPF problem. This paper shows a formulation of the PPF problem under a Bayesian inference perspective and also presents an approximate Bayesian inference method as a suitable solution of a PPF problem. The proposed method assumes a solution drew from a prior distribution, then it obtains simulated data (active and reactive power injected) using power flow equations and finally compares the observed data and simulated data for accepting the solution or rejecting these variables. This overall procedure is known as Approximate Bayesian Computation (ABC). An experimental comparison between the proposed methodology and traditional Monte Carlo simulation is also shown. The proposed methods have been applied on a 6 bus test system and 39 bus test system modified to include a wind farm. Results show that the proposed methodology based on ABC is another alternative for solving the probabilistic power flow problem; similarly this approximate method take less computation time for obtaining the probabilistic solution with respect to the state-of-the-art methodologies.

1 Introduction

A power flow study is an essential tool for the analysis, operation and planning of a power system (PS). When the power generation, loads and the network configuration are specified, it is possible to compute the steady-state of the PS. It is common to use constant parameters to solve the power flow problem (Aien et al., 2014). However, in a real PS, the load demand, network configuration and power supplies are not fixed, due to uncertainty of customer load demand, uncertainty of equipment failure and operation, and uncertainty of weather. Therefore, a deterministic power flow study does not compute correctly the state of the system (Soleimanpour and Mohammadi, 2013; Sansavini et al., 2014).
A probabilistic power flow (PPF) analysis assumes that specific variables in the system can be treated as random variables with particular probability distributions, where the PPF goal is to obtain probability distributions over voltages, angles and power flows

between lines. The solution methods for PPF problem can be classified into two main categories: simulation-based methods or analytical methods (Aien et al., 2014). In the first category, the most widely used method is Monte Carlo simulation (MCS). This method uses repeated deterministic solutions when the input random variables (loads, power generation and voltages at PV nodes) follow an associated distribution function that accounts for the uncertainty. The second category includes approaches based on probabilistic analysis intervals (Briceno et al., 2012), cumulant methods (Le et al., 2013) and point estimation methods (Su, 2005). These methods are computationally more effective than methods based on simulation, however they require mathematical assumptions or approximations for feasible solutions (Soleimanpour and Mohammadi, 2013). Hence, the analytical methods may offer less accurate solutions than approaches based on simulation methods (Soleimanpour and Mohammadi, 2013).

In this paper, we use simulation-based methods under a Bayesian inference perspective. We assume that the state variables (angles at PQ and PV nodes, and voltages at PQ nodes) have a prior distribution, and we want to obtain the posterior distribution for these variables. From the point of view of Bayesian inference, it is necessary to specify a prior distribution of state variables and a likelihood function for the PPF problem, and then using the Bayes theorem, the posterior distribution for these variables can be computed.

Different approaches have used Bayesian inference applied to power systems. In Carmona-Delgado et al. (2015), the authors model the input random variables as multivariate Gaussian mixture distributions and use the expectation maximization algorithm for obtaining samples from these mixture distributions and then, they use these random samples configuration and deterministic optimization for calculating the probability distributions of state variables. Another study uses a kernel density estimation for inferring the probability distribution of wind speed and explores the impacts of high dimensional dependences of wind speed among wind farms on the PS (Cao and Yan, 2017). However, they do not use a likelihood function of the powers injected to the PS given the state variables. Since the likelihood function for the PPF problem has not been defined, we need either to propose a likelihood function or to use likelihood-free Monte Carlo approaches. In this paper, we use likelihood-free inference. Within of likelihood-free methods, Approximate Bayesian Computation (ABC) can be employed to infer posterior distributions without having to evaluate likelihood functions (Pritchard et al., 1999; Wilkinson, 2013).

We evaluate the performance of two likelihood-free algorithms, namely, ABC and ABC MCMC. ABC is based on rejection sampling (Wilkinson, 2013). We also use ABC MCMC which it combines the ABC approach and the Markov Chain Monte Carlo (MCMC) method (Marjoram et al., 2003). We apply the ABC methods and MCS over 2 test systems: the IEEE 6-bus test system and a 39-bus test system modified.

2 Power flow analysis

According to Su (2005), the power flow analysis can be expressed by two sets of nonlinear equations. Given the network configuration, the power flow equations can be written as follows,

$$\mathbf{b} = \mathbf{g}\left(\mathbf{x}\right), \tag{1}$$

$$\mathbf{z} = \mathbf{h}\left(\mathbf{x}\right), \tag{2}$$

where $\mathbf{g}$ and $\mathbf{h}$ are nonlinear power flow equations; $\mathbf{x}$ is a state variable vector that contains the angles at the PQ and PV nodes, and the voltages at the PQ nodes; $\mathbf{b}$ is a vector with entries given by the net active power and the net reactive power injected, which are known. $\mathbf{z}$ is a vector which the elements are the power flows through lines. To solve the power flow problem (obtain $\mathbf{x}$ using Eq. (1)), it is common to use the Gauss-Seidel method or the Newton-Raphson method iteratively (Wood and Wollenberg, 1996), then it is possible to compute the power flow employing Eq. (2).
We propose to use Eq. (1) for obtaining an approximation of the probability distribution over the voltages and angles, when the input data is modeled by combinations of input random variables. The goal of our approach is to use a Bayesian perspective for inferring a probability distribution over $\mathbf{x}$ given observed data or powers injected considering uncertainty. In our context, and using Bayes theorem,

$$p\left(\mathbf{x}|\,\mathcal{D}\right) = \frac{p\left(\mathcal{D}|\,\mathbf{x}\right)p\left(\mathbf{x}\right)}{p\left(\mathcal{D}\right)}, \tag{3}$$

where $p\left(\mathbf{x}\right)$ is the prior distribution (prior) for $\mathbf{x}$, $p\left(\mathcal{D}|\,\mathbf{x}\right)$ is the likelihood function, and $p\left(\mathbf{x}|\,\mathcal{D}\right)$ is the posterior probability distribution (posterior) of the state variable $\mathbf{x}$ given the observed data $\mathcal{D}$. The posterior quantifies the knowledge about the unknown variables, and evaluate the uncertainty in $\mathbf{x}$ after observing $\mathcal{D}$ (Murphy, 2012). The term $p\left(\mathcal{D}\right)$ is a normalization constant, and it is given by (Murphy, 2012),

$$p\left(\mathcal{D}\right) = \int p\left(\mathcal{D}|\,\mathbf{x}\right)p\left(\mathbf{x}\right)d\mathbf{x}. \tag{4}$$

From Eq. (3), posterior, and Eq. (4), evidence, depend on a likelihood function, and as we mentioned above, the likelihood function for PPF problem has not been defined, therefore we can not compute an analytical solution in a closed-form for the posterior and the evidence. For this reason, we need either to construct a likelihood function or to work with likelihood-free methods for obtaining the posterior. In this paper, we employ likelihood-free inference or Approximate Bayesian Computation methods, that consists in generating samples (powers injected) through a simulator. For the PPF problem, this simulator is given by expression $\mathbf{g}\left(\mathbf{x}\right)$ in Eq. (1).

3 Materials and methods

IEEE 6-bus test system For our experiments, we use the IEEE 6-bus test system shown in Su (2005) (Fig. 2). From Su (2005), we employ the same nodal data, and consider the transmission line parameters without uncertainty. Specifically, we model active and reactive power demands through Gaussian distributions. For active power generation at buses 2 and 3, we use Gaussian distributions. For the voltage at PV nodes, that is, voltages at buses 2 and 3, we also employ Gaussian distributions.

IEEE 39-bus test system modified As part of our experiments, we also use the IEEE 39-bus test system. [1] In this case study, we modeled all input random variables (real and reactive demanded power at PQ nodes; voltage magnitudes and real power injected at PV nodes) from Gaussian distributions; and add a wind farm at bus 32, similar to Soleimanpour and Mohammadi (2013), where the output power from one wind turbine is given by

$$P_w(v_w) = \begin{cases} 0 & v_w \leq v_{cin}, \\ 0.5\rho A_w C_p v_w^3 & v_{cin} < v_w \leq v_r, \\ P_r & v_{cout} < v_w, \end{cases} \tag{5}$$

where ρ is the air density; $A_w = \pi R^2$ is the area of the wind turbine rotor; R is the radius of the rotor; C_p is a coefficient of power; v_{cin} is the cut-in wind speed, at which the wind turbine generator starts generating power (Sansavini et al., 2014); v_{cout} is cut-out wind speed, at which the wind turbine generator is shut down for safety reasons (Sansavini et al., 2014); v_r is the nominal rotational speed; P_r is the nominal wind power; and v_w is the wind speed that is assumed to follow a Weibull distribution (Soleimanpour and Mohammadi, 2013),

$$f(v_w | a, b) = \frac{b}{a}\left(\frac{v_w}{a}\right)^{b-1} e^{-\left(\frac{v_w}{a}\right)^b}, \tag{6}$$

where a and b are the scale and shape parameters, respectively. For our experiments, we use $a = 15$ and $b = 2.5$. For the parameters in Eq. (5), we use $v_{cin} = 3$ m/s, $v_{cout} = 25$m/s, $v_r = 10.28$m/s, $C_p = 0.473$ and $R = 45$m, We adopt similar parameters to the ones used by Soleimanpour and Mohammadi (2013).

In this paper, we also model the wind farm output power as a Gaussian random variable, similar to Sansavini et al. (2014), the output power can be modeled as: $P_{wt}(v_w) = P_w(v_w) + \omega$, where $P_{wt}(v_w)$ is the wind farm output power; $P_w(v_w)$ is the deterministic output power expressed by Eq. (5); and ω is a white Gaussian noise, that is, ω is modeled using a Gaussian distribution with zero mean value and variance $\sigma_\omega^2 = 0.000001$. The output power from the wind farm is shown in Fig. 1.

Approximate Bayesian Computation Methods In this paper, we use likelihood-free Monte Carlo approaches, specifically Approximate Bayesian Computation (ABC) methods, as an alternative for solving the PPF problem. Given a prior distribution $p(\mathbf{x})$, the goal in ABC is to approximate the posterior distribution using the following model,

$$p(\mathbf{x}| \mathcal{D}) \propto g(\mathcal{D}| \mathbf{x}) p(\mathbf{x}), \tag{7}$$

where $g(\cdot)$ is a function that depends on the model. These likelihood-free algorithms replace the computation of the likelihood with a comparison between summary statistics

[1] This systems is available at `http://www.pserc.cornell.edu/matpower/`

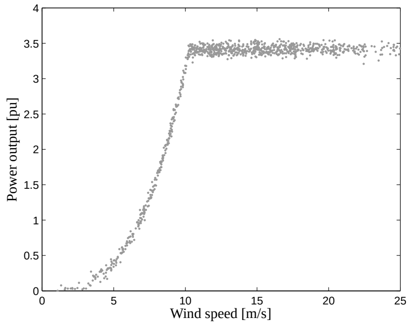


Fig. 1: Output power from wind generator connected at node 32.

of the observed data (active and reactive power injected) and summary statistics from simulated data. The most basic ABC algorithm is based on rejection sampling, and it is given by (Wilkinson, 2013)

Algorithm 1: ABC rejection

Draw $\mathbf{x}$ from $p(\mathbf{x})$
Simulate $\mathcal{D}'$ using $g(\cdot|\,\mathbf{x})$
Accept $\mathbf{x}$ if $\rho(s(\mathcal{D}), s(\mathcal{D}')) \leq \epsilon$

where $\rho(\cdot)$ is a distance measure; ϵ is a tolerance that determines the accuracy of the algorithm; $s(\mathcal{D})$ are the summary statistics from observed data, and $s(\mathcal{D}')$ are summary statistics from simulated data. For our study, $\mathcal{D}$ are injections of reactive and active power. It is important to mention that the empirical distribution over accepted samples for $\mathbf{x}$ is an approximation to the true posterior distribution, the approximation can be expressed through $p(\mathbf{x}|\,\rho(s(\mathcal{D}), s(\mathcal{D}')) \leq \epsilon)$. If $\epsilon = 0$, the samples that we draw will come from the true posterior, however the algorithm would need to perform more simulations for accepting any sample (Wilkinson, 2013). Since choosing $\epsilon = 0$ would be prohibitively expensive, the value of ϵ that we choose will affect the quality of the approximation. From algorithm 1, another issue to address how to choose a suitable distance measure or the summary statistics. See Wilkinson (2013) for details. To avoid some of these problems, several ABC approaches have been proposed, like ABC MCMC (Marjoram et al., 2003), that is described below.

ABC MCMC In the algorithm 1, it is possible to have low acceptance rates when the prior distribution is not close to the posterior distribution. An alternative solution for this problem is provided by a MCMC approach (Marjoram et al., 2003). ABC MCMC proposes a new parameter value using a proposal distribution ($q(\cdot)$). This distribution must be chosen so that we can easily evaluate it, and generate samples from it. For more information about the proposal distribution and its parameters, see Murphy (2012), chapter 24. Algorithm 2 shows how the ABC MCMC method can be implemented.
Algorithm 2 also obtains samples from an approximated posterior over $\mathbf{x}$ (Toni et al., 2009). However, according to Toni et al. (2009), ABC MCMC may get stuck in regions

Algorithm 2: ABC MCMC

Initialize $\mathbf{x}_0$
for $i = 1, \ldots, N_s$ **do**
  Draw $\mathbf{x}^*$ from $q\left(\mathbf{x}|\, \mathbf{x}_i\right)$
  Simulate $\mathcal{D}'$ using $g\left(\cdot|\, \mathbf{x}^*\right)$
  if $\rho\left(s\left(\mathcal{D}\right), s\left(\mathcal{D}'\right)\right) \leq \epsilon$
  Accept $\mathbf{x}_{i+1} = \mathbf{x}^*$ with probability
  $\alpha = \min\left(1, \frac{p\left(\mathbf{x}^*\right) q\left(\mathbf{x}_i|\mathbf{x}^*\right)}{p\left(\mathbf{x}_i\right) q\left(\mathbf{x}^*|\mathbf{x}_i\right)}\right)$
  Otherwise $\mathbf{x}_{i+1} = \mathbf{x}_i$

of low probability for the accepted samples. For avoiding this, we propose to use a multivariate Gaussian distribution as proposal distribution, where its mean value can be computed using power system parameters. We take $\mathbf{x}_0$ as an all-ones vector for voltages and the solution of DC power flow algorithm for angles.

3.1 Classic probabilistic power flow analysis

Classic probabilistic power flow analysis or Monte Carlo simulation (MCS) is based on repetitive solutions to the DLF problem (i.e. $\mathbf{g}\left(\mathbf{x}\right) - \mathbf{b} = \mathbf{0}$ from Eq. (1)) using a set of random vectors drawn from input random variables. In this paper, we use the Newton Raphson method as deterministic method of solution, and use MATPOWER to implement MCS. [2]

3.2 Validation

For validation purposes, we compute the relative error (RE) between the value obtained using power flow analysis without considering uncertainty over variables and the mean value for each variable obtained using the simulation methods. The RE is given by

$$RE = \left\| \frac{x_{wu} - x_{mp}}{x_{wu}} \right\|, \tag{8}$$

where x_{wu} is the original value without considering uncertainty over variables, and x_{mp} is the estimated posterior mean for that variable obtained through each simulation method. This error is only for the IEEE 6-bus test system. For the IEEE-39 mentioned above, we compute relative errors with respect to the values calculated using MCS, that is, we employ (Su, 2005),

$$\varepsilon_x^{\mu} = \left| \frac{\mu_x - \mu_x^*}{\mu_x} \right|, \quad \varepsilon_x^{\sigma} = \left| \frac{\sigma_x - \sigma_x^*}{\sigma_x} \right|, \tag{9}$$

where ε_x^{μ} and ε_x^{σ} are relative error for the mean and standard deviation values; μ_x and σ_x are the mean and standard deviation obtained using MCS; μ_x^* and σ_x^* are the mean and standard deviation computed with the ABC methods exposed above.

[2] This package is available at `http://www.pserc.cornell.edu/matpower/`

3.3 Procedure

For the IEEE 6-bus test system, we model the input random variables as Gaussian distributions. The mean values and standard deviations are assumed as mentioned in Su (2005). We also use Gaussian distributions for the voltages at nodes 2 and 3, which are PV buses; mean values are assumed as in Su (2005); and the standard deviations values are set as $\sigma_v = 0.000001$, since Su (2005) did not model these variables as random variables.

For the IEEE 39-bus test system, the input random variables are modeled by Gaussian distributions and a wind farm at bus 32 was added, which includes 2134 wind turbines and the output power from a wind farm was shown in Fig. 1.

From the probability distributions characterizing the above random variables, we generate 1000 samples for each variable for all test systems. We then apply the MCS for each sample configuration. Using this set of samples, we compute $\mathbf{b}_i$ (see Eq. (1)), which is used as observed data for the ABC methods. We then apply ABC and ABC MCMC.

For ABC and ABC MCMC, we use $\epsilon = (0.04, 0.3)$ in the IEEE 6-bus and 39-bus test system, respectively. For both methods, we use a multivariate Gaussian distribution as prior distribution for the voltages, where its mean vector is an all-ones vector, and the covariance matrix is diagonal with parameters σ_V^2. We use the DC power flow algorithm (Wood and Wollenberg, 1996), and employ an uniform distribution for each angle as prior distribution for them: we first apply the DC power flow algorithm to the system, from this solution, an uniform distribution is defined ($\theta_i \sim \mathcal{U}(a_i, b_i)$) for each angle. The parameters a_i and b_i will be computed as $a_i = \theta_i^{DC} - \Delta\theta$ and $b_i = \theta_i^{DC} + \Delta\theta$. $\Delta\theta$ quantifies the error arising from the solution obtained by DC power flow algorithm with respect to AC power flow solution (Eq. (1)). We choose $\Delta\theta$ equal to 0.07.

For ABC and ABC MCMC, the simulation function was set as $\mathbf{g}(\mathbf{x})$ (see Eq. (1)). The distance measure that we use is given by

$$\rho\left(s\left(\mathcal{D}\right), s\left(\mathcal{D}'\right)\right) = \sqrt{\frac{1}{M}\left(s\left(\mathbf{b}\right) - s\left(\mathbf{g}\left(\mathbf{x}\right)\right)\right)^{\top}\left(s\left(\mathbf{b}\right) - s\left(\mathbf{g}\left(\mathbf{x}\right)\right)\right)}, \qquad (10)$$

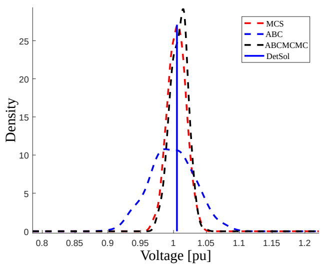
where M is the number of injected powers (active and reactive powers). For choosing s, we use a surrogate modeling between $\mathbf{b}$ and $\mathbf{x}$, specifically, we use a feedforward neural network for learning the summary statistics using the approach shown in Fearnhead and Prangle (2012). We use 10 neurons in the hidden layer, employ $\mathbf{b}$ and the mean of solutions obtained by MCS as training data, for both systems.

4 Results and discussion

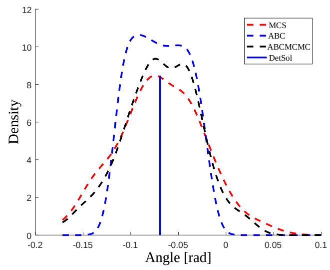
In this section, we present a comparison among ABC, ABC MCMC and MCS for solving the PPF problem using the IEEE 6-bus test system and IEEE 39-bus test system mentioned in section 3.

4.1 Results from IEEE 6-bus test system

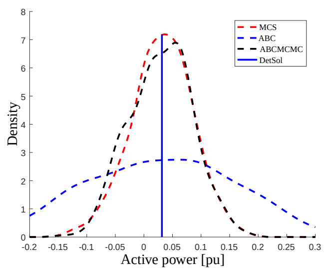
ABC, ABC MCMC and MCS were applied over the IEEE 6-bus test system, where the goal is to obtain the posterior for $\mathbf{x} = \left[\theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6\ V_4\ V_5\ V_6\right]^\top$, having 1000 samples from the input random variables, i.e. with 1000 different $\mathbf{b}_i$ or $\mathcal{D}_i$ vectors. In Fig. 2, we show the posterior for θ_6, V_6, active and reactive power flow between nodes 5 and 6 (P_{56} and Q_{56}), respectively.



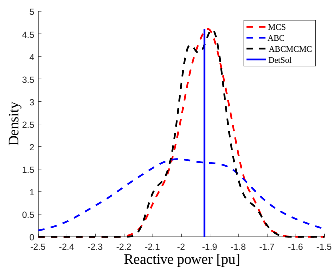
(a) Posterior for V_6



(b) Posterior for θ_6



(c) Posterior for P_{56}



(d) Posterior for Q_{56}

Fig. 2: Posteriors for V_6, θ_6, P_{56} and Q_{56} using ABC ABC MCMC and MCS. The dashed red, blue and black lines are the responses obtained by MCS, ABC and ABC MCMC. The blue vertical solid line is the deterministic solution.

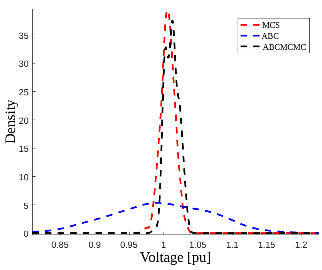
Notice that the means of the posteriors for the different variables are close to the deterministic solution. The posteriors obtained by ABC MCMC are close to the posteriors using MCS. Since all input random variables are modeled by Gaussian distributions, we can compare the deterministic solution (see the vertical lines of the above figures) of the system and estimated posterior mean using the methods exposed when considering the relative errors (RE) shown in Eq. (8). In table 1, we show Relative errors for voltages (RE_v) and angles (RE_θ) obtained by MCS, ABC and ABC MCMC. We also present the computation times (CompTime) took by MCS, ABC and ABC MCMC for solving PPF problem using the 6-bus test system. All simulations have been performed on an Intel Core i7 PC with a 2.1GHz processor. From table 1, notice that MCS obtained the least RE_v, however this method took the highest CompTime for solving this PPF problem. We note that ABC MCMC took the least CompTime for analyzing this system, and obtained the least RE_θ. Notice that ABC is the least efficient method.

Method	$RE_v\,[\%]$	$RE_\theta\,[\%]$	CompTime $[s]$
MCS	**0.0558**	1.4478	21.902
ABC	1.2179	16.337	38.173
ABC MCMC	0.3147	**0.8544**	**17.347**

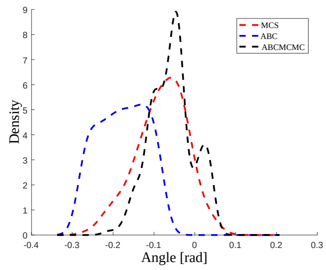
Table 1: RE for voltages (RE_v) and angles (RE_θ) obtained by each method. CompTime is the computation time took by all methods for solving PPF problem using the 6-bus system.

4.2 Results from IEEE 39-bus test system

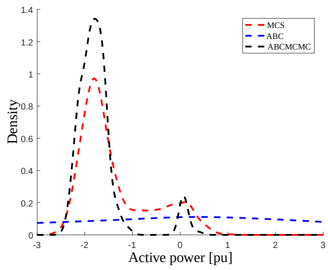
After showing the results using the 6-bus system, we proceed to present the comparison of results obtained by ABC and ABC MCMC with respect to the results employing MCS, when the dimension of the unknown variables is increased.



(a) Posterior for V_{15}.



(b) Posterior for θ_{15}.



(c) Posterior for P_{39-9}

Fig. 3: Posteriors for θ_{15}, V_{15} and P_{39-9} using ABC ABC MCMC and MCS. The dashed red, blue and black lines are the responses obtained by MCS, ABC and ABC MCMC.

We use the 39-bus test system, with 67 variables: 38 angles and 29 voltages. In Fig. 3, we present the posterior of θ_{15}, V_{15} and P_{39-9} using all methods exposed. Notice that in this case the ABC does not estimate correctly the posterior of θ_{15}, V_{15} and P_{39-9}. In contrast, the posteriors obtained by ABC MCMC are close to the posteriors using MCS. Since we do not have a ground-truth solution, we use estimated posterior mean and standard deviation (for angles and voltages) of MCS as references values. Using the Eq. (9) and the mean angle values, ABC obtained $\varepsilon_\theta^\mu = 295.12\%$ and ABC MCMC presents $\varepsilon_\theta^\mu = 67.912\%$, and for standard deviation, ABC computed $\varepsilon_\theta^\sigma = 17.345\%$ and ABC MCMC presents $\varepsilon_\theta^\sigma = 18.835\%$. For voltages, ABC obtained a $\varepsilon_V^\mu = 2.5157\%$ and a $\varepsilon_\theta^\sigma = 499.83\%$, ABC MCMC computed a $\varepsilon_V^\mu = 0.5425\%$ and a $\varepsilon_\theta^\sigma = 8.4428\%$. Notice that ABC MCMC obtained the best results. ABC suffers when the dimension of the unknown variables is increased. Finally, MCS took 24.211s for solving the PPF problem, ABC MCMC employed 18.361s and ABC took 35.895s.

5 Conclusions

We introduced a new alternative for solving the PPF problem, using approximate Bayesian computation methods. We demonstrated that ABC can work for small system using Gaussian random variables. We also showed that the posteriors of the sate variables obtained by ABC MCMC are close to the results using MCS, similarly this approximate method take less computation time for obtaining the probabilistic solution with respect to MCS.

Acknowledgements C. D. Zuluaga is being funded by Department of Science, Technology and Innovation, Colciencias. This work was developed within the research project: "Approximate Bayesian Computation applied to probabilistic power flow" financed by Universidad Tecnológica de Pereira, Colombia.

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